MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

62nd Annual

AMC 12 B

American Mathematics Contest 12 B

Wednesday, February 23, 2011

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 29th annual American Invitational Mathematics Examination (AIME) on Thursday, March 17, 2011 or Wednesday, March 30, 2011. More details about the AIME and other information are on the back page of this test booklet.

2011

AMC 12 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 23, 2011

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 23, 2011. Nothing is needed from inside this package until February 23.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.

3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.

4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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1. What is
$$\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}\ ?$$

(A) -1 **(B)** $\frac{5}{36}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{147}{60}$ **(E)** $\frac{43}{3}$

2. Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

(A) 80 **(B)** 82 **(C)** 85 **(D)** 90 **(E)** 95

3. LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

(A) $\frac{A+B}{2}$ **(B)** $\frac{A-B}{2}$ **(C)** $\frac{B-A}{2}$ **(D)** $B-A$ **(E)** $A+B$

4. In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

(A) 116 **(B)** 161 **(C)** 204 **(D)** 214 **(E)** 224

5. Let N be the second smallest positive integer that is divisible by every positive integer less than 7. What is the sum of the digits of N?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

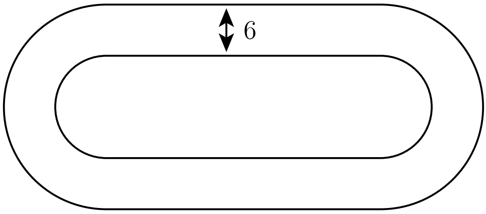
6. Two tangents to a circle are drawn from a point A. The points of contact B and C divide the circle into arcs with lengths in the ratio $2:3$. What is the degree measure of $\angle BAC$?

(A) 24 **(B)** 30 **(C)** 36 **(D)** 48 **(E)** 60

7. Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?

(A) 3 **(B)** $\frac{33}{7}$ **(C)** $\frac{39}{7}$ **(D)** 9 **(E)** $\frac{99}{10}$

8. Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?



(A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

9. Two real numbers are selected independently at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

(A) $\frac{1}{9}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

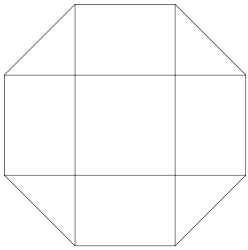
10. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

11. A frog located at (x, y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0, 0)$ and ends at $(1, 0)$. What is the smallest possible number of jumps the frog makes?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2-\sqrt{2}$

13. Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

14. A segment through the focus F of a parabola with vertex V is perpendicular to $\overline{FV}$ and intersects the parabola in points A and B. What is $\cos(\angle AVB)$?

(A) $-\frac{3\sqrt{5}}{7}$ **(B)** $-\frac{2\sqrt{5}}{5}$ **(C)** $-\frac{4}{5}$ **(D)** $-\frac{3}{5}$ **(E)** $-\frac{1}{2}$

15. How many positive two-digit integers are factors of $2^{24}-1$?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

16. Rhombus $ABCD$ has side length 2 and $\angle B = 120°$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

17. Let $f(x) = 10^{10x}$, $g(x) = \log_{10}(\frac{x}{10})$, $h_1(x) = g(f(x))$, and $h_n(x) = h_1(h_{n-1}(x))$ for integers $n \geq 2$. What is the sum of the digits of $h_{2011}(1)$?

(A) 16,081 **(B)** 16,089 **(C)** 18,089 **(D)** 18,098 **(E)** 18,099

18. A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

(A) $5\sqrt{2}-7$ **(B)** $7-4\sqrt{3}$ **(C)** $\frac{2\sqrt{2}}{27}$ **(D)** $\frac{\sqrt{2}}{9}$ **(E)** $\frac{\sqrt{3}}{9}$

19. A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \le 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

(A) $\frac{51}{101}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{51}{100}$ **(D)** $\frac{52}{101}$ **(E)** $\frac{13}{25}$

20. Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of $\triangle BDE$ and $\triangle CEF$. What is $XA + XB + XC$?

(A) 24 **(B)** $14\sqrt{3}$ **(C)** $\frac{195}{8}$ **(D)** $\frac{129\sqrt{7}}{14}$ **(E)** $\frac{69\sqrt{2}}{4}$

21. The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x - y|$?

(A) 24 **(B)** 48 **(C)** 54 **(D)** 66 **(E)** 70

22. Let T_1 be a triangle with sides 2011, 2012, and 2013. For $n \ge 1$, if $T_n = \triangle ABC$ and D, E, and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$

23. A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

(A) 161 **(B)** 185 **(C)** 195 **(D)** 227 **(E)** 255

24. Let $P(z) = z^8 + (4\sqrt{3}+6)z^4 - (4\sqrt{3}+7)$. What is the minimum perimeter among all the 8-sided polygons in the complex plane whose vertices are precisely the zeros of $P(z)$?

(A) $4\sqrt{3}+4$ **(B)** $8\sqrt{2}$ **(C)** $3\sqrt{2}+3\sqrt{6}$ **(D)** $4\sqrt{2}+4\sqrt{3}$ **(E)** $4\sqrt{3}+6$

25. For every m and k integers with k odd, denote by $\left[\frac{m}{k}\right]$ the integer closest to $\frac{m}{k}$. For every odd integer k, let $P(k)$ be the probability that

$$\left[\frac{n}{k}\right] + \left[\frac{100-n}{k}\right] = \left[\frac{100}{k}\right]$$

for an integer n randomly chosen from the interval $1 \le n \le 99!$. What is the minimum possible value of $P(k)$ over the odd integers k in the interval $1 \le k \le 99$?

(A) $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{44}{87}$ **(D)** $\frac{34}{67}$ **(E)** $\frac{7}{13}$

MATHEMATICAL ASSOCIATION OF AMERICA

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

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2011 AIME

The 29th annual AIME will be held on Thursday, March 17, with the alternate on Wednesday, March 30. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 40th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2011. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org