MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

62nd Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 8, 2011

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 29th annual American Invitational Mathematics Examination (AIME) on Thursday, March 17, 2011 or Wednesday, March 30, 2011. More details about the AIME and other information are on the back page of this test booklet.

2011

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 8, 2011

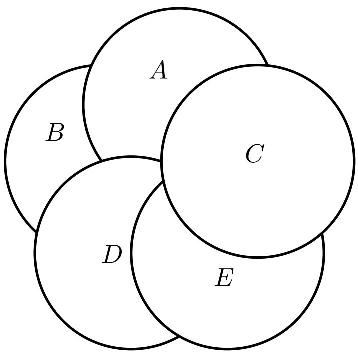
****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 8, 2011. Nothing is needed from inside this package until February 8.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

1. A cell phone plan costs \$20 each month, plus 5¢ per text message sent, plus 10¢ for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

(A) \$24.00 **(B)** \$24.50 **(C)** \$25.50 **(D)** \$28.00 **(E)** \$30.00

2. There are 5 coins placed flat on a table according to the figure. What is the order of the coins from top to bottom?



(A) (C, A, E, D, B) **(B)** (C, A, D, E, B) **(C)** (C, D, E, A, B)
(D) (C, E, A, D, B) **(E)** (C, E, D, A, B)

3. A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

4. At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

(A) 12 **(B)** $\frac{37}{3}$ **(C)** $\frac{88}{7}$ **(D)** 13 **(E)** 14

5. Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

(A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

6. The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

7. A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was \$17.71. What was the cost of a pencil in cents?

(A) 7 **(B)** 11 **(C)** 17 **(D)** 23 **(E)** 77

8. In the eight-term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

(A) 17 **(B)** 18 **(C)** 25 **(D)** 26 **(E)** 43

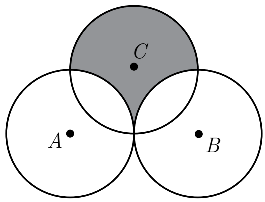
9. At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her sibling and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

(A) 324 **(B)** 441 **(C)** 630 **(D)** 648 **(E)** 882

10. A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{18}$

11. Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?



(A) $3 - \frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\frac{3\pi}{4}$ **(E)** $1 + \frac{\pi}{2}$

12. A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock A. How many hours did it take the power boat to go from A to B?

(A) 3 **(B)** 3.5 **(C)** 4 **(D)** 4.5 **(E)** 5

13. Triangle ABC has side-lengths $AB = 12$, $BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 36 **(E)** 42

14. Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

(A) $\frac{11}{81}$ **(B)** $\frac{13}{81}$ **(C)** $\frac{5}{27}$ **(D)** $\frac{17}{81}$ **(E)** $\frac{19}{81}$

15. The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

(A) $3\sqrt{2}$ **(B)** $\frac{13}{3}$ **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $\frac{13}{2}$

16. Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

17. Circles with radii 1, 2, and 3 are mutually externally tangent. What is the area of the triangle determined by the points of tangency?

(A) $\frac{3}{5}$ **(B)** $\frac{4}{5}$ **(C)** 1 **(D)** $\frac{6}{5}$ **(E)** $\frac{4}{3}$

18. Suppose that $|x+y|+|x-y|=2$. What is the maximum possible value of x^2-6x+y^2?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

19. At a competition with N players, the number of players given elite status is equal to

$$2^{1+\lfloor \log_2(N-1) \rfloor} - N.$$

Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N?

Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

(A) 38 **(B)** 90 **(C)** 154 **(D)** 406 **(E)** 1024

20. Let $f(x)=ax^2+bx+c$, where a, b, and c are integers. Suppose that $f(1)=0$, $50<f(7)<60$, $70<f(8)<80$, and $5000k<f(100)<5000(k+1)$ for some integer k. What is k?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

21. Let $f_1(x)=\sqrt{1-x}$, and for integers $n \geq 2$, let $f_n(x)=f_{n-1}(\sqrt{n^2-x})$. If N is the largest value of n for which the domain of f_n is nonempty, the domain of f_N is $\{c\}$. What is $N+c$?

(A) -226 **(B)** -144 **(C)** -20 **(D)** 20 **(E)** 144

22. Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

(A) 1500 **(B)** 1560 **(C)** 2320 **(D)** 2480 **(E)** 2500

23. Let $f(z) = \frac{z+a}{z+b}$ and $g(z) = f(f(z))$, where a and b are complex numbers. Suppose that $|a| = 1$ and $g(g(z)) = z$ for all z for which $g(g(z))$ is defined. What is the difference between the largest and smallest possible values of $|b|$?

(A) 0 **(B)** $\sqrt{2} - 1$ **(C)** $\sqrt{3} - 1$ **(D)** 1 **(E)** 2

24. Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, and $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

(A) $\sqrt{15}$ **(B)** $\sqrt{21}$ **(C)** $2\sqrt{6}$ **(D)** 5 **(E)** $2\sqrt{7}$

25. Triangle ABC has $\angle BAC = 60°$, $\angle CBA \leq 90°$, $BC = 1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of the pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

(A) $60°$ **(B)** $72°$ **(C)** $75°$ **(D)** $80°$ **(E)** $90°$

MAA

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12
and orders for publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

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2011 AIME

The 29th annual AIME will be held on Thursday, March 17, with the alternate on Wednesday, March 30. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 40th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2011. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org