MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

12th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 8, 2011

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 29th annual American Invitational Mathematics Examination (AIME) on Thursday, March 17, 2011 or Wednesday, March 30, 2011. More details about the AIME and other information are on the back page of this test booklet.

2011

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 8, 2011

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 8, 2011. Nothing is needed from inside this package until February 8.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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1. A cell phone plan costs \$20 each month, plus 5¢ per text message sent, plus 10¢ for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

   (A) \$24.00 **(B)** \$24.50 **(C)** \$25.50 **(D)** \$28.00 **(E)** \$30.00

2. A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

3. Suppose $[a\ b]$ denotes the average of a and b, and $\{a\ b\ c\}$ denotes the average of a, b, and c. What is $\{\{1\ 1\ 0\}\ [0\ 1]\ 0\}$?

   (A) $\frac{2}{9}$ **(B)** $\frac{5}{18}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{7}{18}$ **(E)** $\frac{2}{3}$

4. Let X and Y be the following sums of arithmetic sequences:

   $$X = 10 + 12 + 14 + \cdots + 100,$$
   $$Y = 12 + 14 + 16 + \cdots + 102.$$

   What is the value of $Y - X$?

   (A) 92 **(B)** 98 **(C)** 100 **(D)** 102 **(E)** 112

5. At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

   (A) 12 **(B)** $\frac{37}{3}$ **(C)** $\frac{88}{7}$ **(D)** 13 **(E)** 14

6. Set A has 20 elements, and set B has 15 elements. What is the smallest possible number of elements in $A \cup B$, the union of A and B?

   (A) 5 **(B)** 15 **(C)** 20 **(D)** 35 **(E)** 300

7. Which of the following equations does not have a solution?

   (A) $(x+7)^2 = 0$
   (B) $|-3x| + 5 = 0$
   (C) $\sqrt{-x} - 2 = 0$
   (D) $\sqrt{x} - 8 = 0$
   (E) $|-3x| - 4 = 0$

8. Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

   (A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

9. A rectangular region is bounded by the graphs of the equations $y = a$, $y = -b$, $x = -c$, and $x = d$, where a, b, c, and d are all positive numbers. Which of the following represents the area of this region?

   (A) $ac + ad + bc + bd$ **(B)** $ac - ad + bc - bd$ **(C)** $ac + ad - bc - bd$
   (D) $-ac - ad + bc + bd$ **(E)** $ac - ad - bc + bd$

10. A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was \$17.71. What was the cost of a pencil in cents?

    (A) 7 **(B)** 11 **(C)** 17 **(D)** 23 **(E)** 77

11. Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on $\overline{AB}$ with $AE = 7 \cdot EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

    (A) $\frac{49}{64}$ **(B)** $\frac{25}{32}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{5\sqrt{2}}{8}$ **(E)** $\frac{\sqrt{14}}{4}$

12. The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

    (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

13. How many even integers are there between 200 and 700 whose digits are all different and come from the set $\{1, 2, 5, 7, 8, 9\}$?

(A) 12 **(B)** 20 **(C)** 72 **(D)** 120 **(E)** 200

14. A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{18}$

15. Roy bought a new battery-gasoline hybrid car. On a trip the car ran exclusively on its battery for the first 40 miles, then ran exclusively on gasoline for the rest of the trip, using gasoline at a rate of 0.02 gallons per mile. On the whole trip he averaged 55 miles per gallon. How long was the trip in miles?

(A) 140 **(B)** 240 **(C)** 440 **(D)** 640 **(E)** 840

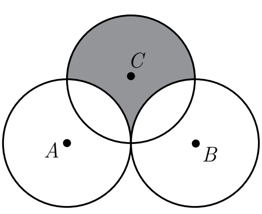
16. Which of the following is equal to $\sqrt{9-6\sqrt{2}}+\sqrt{9+6\sqrt{2}}$?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{6}$ **(C)** $\frac{7\sqrt{2}}{2}$ **(D)** $3\sqrt{3}$ **(E)** 6

17. In the eight-term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

(A) 17 **(B)** 18 **(C)** 25 **(D)** 26 **(E)** 43

18. Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?



(A) $3-\frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\frac{3\pi}{4}$ **(E)** $1+\frac{\pi}{2}$

19. In 1991 the population of a town was a perfect square. Ten years later, after an increase of 150 people, the population was 9 more than a perfect square. Now, in 2011, with an increase of another 150 people, the population is once again a perfect square. Which of the following is closest to the percent growth of the town's population during this twenty-year period?

(A) 42 **(B)** 47 **(C)** 52 **(D)** 57 **(E)** 62

20. Two points on the circumference of a circle of radius r are selected independently and at random. From each point a chord of length r is drawn in a clockwise direction. What is the probability that the two chords intersect?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

21. Two counterfeit coins of equal weight are mixed with 8 identical genuine coins. The weight of each of the counterfeit coins is different from the weight of each of the genuine coins. A pair of coins is selected at random without replacement from the 10 coins. A second pair is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 selected coins are genuine?

(A) $\frac{7}{11}$ **(B)** $\frac{9}{13}$ **(C)** $\frac{11}{15}$ **(D)** $\frac{15}{19}$ **(E)** $\frac{15}{16}$

22. Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

23. Seven students count from 1 to 1000 as follows:

- Alice says all of the numbers, except she skips the middle number in each consecutive group of three numbers. That is, Alice says $1, 3, 4, 6, 7, 9, \ldots,$ $997, 999, 1000$.
- Barbara says all of the numbers that Alice doesn't say, except she also skips the middle number in each consecutive group of three numbers.
- Candice says all of the numbers that neither Alice nor Barbara says, except she also skips the middle number in each consecutive group of three numbers.
- Debbie, Eliza, and Fatima say all of the numbers that none of the students with first names beginning before theirs in the alphabet say, except each also skips the middle number in each of her consecutive groups of three numbers.
- Finally, George says the only number that no one else says.

What number does George say?

(A) 37 **(B)** 242 **(C)** 365 **(D)** 728 **(E)** 998

24. Two distinct regular tetrahedra have all their vertices among the vertices of the same unit cube. What is the volume of the region formed by the intersection of the tetrahedra?

(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{2}}{12}$ **(C)** $\frac{\sqrt{3}}{12}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{\sqrt{2}}{6}$

25. Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

(A) 1500 **(B)** 1560 **(C)** 2320 **(D)** 2480 **(E)** 2500

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

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2011 AIME

The 29th annual AIME will be held on Thursday, March 17, with the alternate on Wednesday, March 30. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 40th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2011. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
amc.maa.org