## Geometry CC WS 2.4 - Composition of transformations

Combining two or more transformations to form a new transformation is called a **composition of transformations**. It can be thought of as sequence of transformations – the image of the first transformation is used as the pre-image for the second and so on.

In the following diagram what rigid motions map  $\Delta ABC$  onto  $\Delta RST$ ?



In the above composition,  $\Delta ABC$  is first translated then reflected.

The symbol for a composition of transformations is an open circle and is performed from RIGHT to LEFT. The example above might be represented as:  $r_{x-axis} \circ T_{-6,0}(\Delta ABC)$ 

Examples: State the coordinates of the point after the indicated composition.

- 1.  $r_{x-axis} \circ T_{2,-3}(4,2)$
- 2.  $T_{-2,5} \circ R_{0,90^{\circ}}(4,2)$
- 3. Find the image of A(4, 2) after the following transformations
  - a.  $T_{1,4} \circ r_{y-axis}(4,2)$
  - b.  $r_{y-axis} \circ T_{1,4}(4,2)$
  - c. Are the two transformations from parts *a* and *b* equivalent (does  $T_{1,4} \circ r_{y-axis} = r_{y-axis} \circ T_{1,4}$ )?

## Composition of transformations is not commutative.

- 4. Find the image of A(4, 2) after the following transformations
  - a.  $R_{0,90^{\circ}} \circ R_{0,180^{\circ}}(4,2)$
  - b.  $R_{0,270^{\circ}}(4,2)$
  - c. Are the two transformations from parts *a* and *b* equivalent?

The composition of two rotations about the same center of rotation can be represented as a single rotation (the sum of the two rotations).  $R_{0,90^{\circ}} \circ R_{0,180^{\circ}}(4,2) = R_{0,270^{\circ}}(4,2)$ 

5. The following composition is represented in the diagram below:  $r_{y=-2} \circ r_{y=4}(\Delta ABC)$ 



Write a single transformation that would map  $\triangle ABC$  onto  $\triangle A''B''C''$ : \_\_\_\_\_\_ A composition of reflections over two parallel lines is equivalent to a translation.

6. The following composition is represented in the diagram below:  $T_{5,-4} \circ T_{6,3}(\Delta ABC)$ 



Write a single transformation that would map  $\triangle ABC$  onto  $\triangle A''B''C''$ :

A composition of two or more translations can be rewritten as a single translation.  $T_{5,-4} \circ T_{6,3} = T_{11,-1}$ 

The composition of reflections over two intersecting lines is equivalent to a rotation.

A glide reflection is the composite transformation in which a figure is reflected in a line and is then translated parallel to the reflecting line.

Any translation or rotation can be expressed as the composition of two reflections.

## **Homework Exercises**

- 1. What is the image of point P(3, 1) under  $r_{x-axis} \circ R_{0,90^{\circ}}$ ?
- 2. What are the coordinates of A', the image of point A(-4, 1) after the composite transformation  $R_{0,90^{\circ}} \circ r_{y=x}$ ?
- 3. The coordinates of  $\Delta JRB$  are J(1, -2), R(-3, 6), and B(4, 5). What are the coordinates of the vertices of its image after the transformation  $T_{5,-4} \circ r_{y-axis}$ ?
- 4. Find the coordinates of  $(R_{90^{\circ}} \circ R_{180^{\circ}})(P)$  if the coordinates of point *P* are (2, -3).
- 5. Find the coordinates of  $r_{y-axis} \circ r_{y=x}(A)$  if the coordinates of A are (6, 2)
- 6. What is the image of P(-4, 7) under the composition  $r_{x=2} \circ r_{y-axis}$ ?
- 7. Find the coordinates of the image of (2, 4) under the transformation  $r_{y-axis} \circ T_{5,-3}$
- 8. Given the transformations:  $B(x, y) \rightarrow (-x, y)$

 $R(x,y) \to (y,x)$ 

What is  $(B \circ R)(5, -1)$ ?

9. Circle the image of **A** under the transformation  $r_{x-axis} \circ (R_{90^\circ})$ 



10. What is the image of *P*(5, 1) under the composition  $r_{x=2} \circ r_{y=x}$ ?

11. In the diagram below, congruent figures 1, 2, and 3 are drawn.



Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

- (1) A rotation followed by a translation
- (2) A translation followed by a rotation
- (3) a reflection followed by a translation
- (4) a translation followed by a reflection
- 12. A sequence of transformations maps a rectangle *ABCD* onto rectangle *A"B"C"D"* as shown in the diagram below.



Which sequence of transformations maps rectangle ABCD onto A'B'C'D' and then maps A'B'C'D' onto A''B''C''D''

- (3) A reflection followed by a rotation
- (4) A translation followed by a rotation
- (3) a reflection followed by a translation
- (4) a translation followed by a reflection

13. In the diagram below  $\triangle ABC$  and  $\triangle XYZ$  are graphed.



State the composition mapping  $\Delta ABC$  onto  $\Delta XYZ$ .